

## **Total Efficiency and Output Power of Nd:YAG Laser with Spatial Interaction Efficiency Factor**

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### **ABSTRACT**

Optimum total efficiency and output power were derived for Nd:YAG laser with symmetrical resonator configuration, taking into account a new interaction factor called spatial interaction efficiency factor. Under the assumption of the same pumping power and the same resonator losses, the results interacted a lower output power and higher lasing threshold provided that the mirror reflectivity was optimized as compared with a laser system that did not take the spatial interaction efficiency factor into consideration.

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### **INTRODUCTION**

Many of the references in the fundamental physics are concerned with the development of resonators for the processes which occur when electromagnetic radiation interacts with a laser medium. The construction of a particular resonator needs a prolonged attention in considering and analyzing the geometric design models and other details of the focusing process of radiation field and therefore, each case should be considered individually. This may explain that somewhat empirical nature of analytical studies concerning the parameters, which could influence identification and the beam characterization of the lasing process.

The difficulties arise in considering the mutual interaction of such parameters and the relative importance of each of them.

(Wetter et al., 1998), used an approximation for the beam waist inside the laser rod as function of pump power to obtain an expression for the output power, taking into account the effects of thermally induced lensing in a Nd:YAG resonator. This was done by considering the laser rod as a thin lens, which was induced by thermally generated birefringence of the crystal.

Impressive improvements in the power conversion rate have been by using high reflectance mirror and a stabilized laser source which made it possible to obtain high build-up power in the external output. (Jaewoo NoH, 1997), treated the problem by constructing a frequency stable Nd:YAG laser and studying the second harmonic generation by adjusting the resonator length to give a beam waist of  $20\mu\text{m}$ , which is a little bit larger than the Boyd optical focusing condition and by selecting several resonator lengths and focusing conditions, the experimental results showed that the optimum cold resonator beam waist should be slightly bigger than the theoretical optimal waist for the most efficient power conversion. On the other hand, (Yashkir et al., 1999), demonstrate the optimum conditions of Nd:YAG rod (1.1%  $\text{Nd}^{+3}$  doping, 40 mm length and 2 mm diameter) and studied the dynamics and interaction characteristics of optical parametric Nd:YAG laser passively Q switched by a  $\text{Cr}^4$ :YAG crystal. The result obtained was a high - efficiency optical parametric laser oscillator provided that the laser rod and the passive Q switch at the proper positions along the radiation field mode inside the resonator.

Besides rod laser system (Eicher et al., 1989), derived the efficiencies and output power relations of a slab laser systems as a function of the small - signal gain and the resonator losses, taking into account the beam overlap and beam enlargement due to total internal reflection effect. With the results obtained, a comparison between a slab system and an equivalent rod system was made. It turned out that under the assumption of the same excitation power and equal losses in both systems, a higher slope efficiency can be achieved in slab system.

The present work is concerned with the derivation of relations for the total efficiency and the output power for Nd:YAG laser. This was achieved by taking into account a new efficiency factor designed as  $\eta_6$ , the spatial interaction efficiency SIE factor. It is defined as the ratio of actual mode volume interacting with the laser rod to the total pumped volume of the laser rod. The intention of this paper is to present the theoretical interpretation of suggested new efficiency factor and its influence on the laser characteristics.

### LASER PARAMETERS

The total efficiency of a solid - state laser  $\eta_{\text{tot}}$  can be defined by the product of the excitation efficiency  $\eta_{\text{excit}}$  and the extraction efficiency  $\eta_{\text{extr}}$ . The excitation efficiency describes the pumping arrangement of a solid - state laser whereas the extraction efficiency describes the resonator of the laser system. The excitation efficiency is simply the product of four individual efficiencies of the system (Koechner, 1996).

$$\eta_{\text{excit}} = \eta_1 \eta_2 \eta_3 \eta_4 \dots \dots \dots (1)$$

Where  $\eta_1$  is the ratio of the fluorescence power at threshold to the total absorbed pump power.  $\eta_2$  is the ratio of lamp radiation power within absorption bands of the laser

material to electrical input power  $P_{in}$ ;  $\eta_3$  is the efficiency obtained in transferring the useful radiation power to laser rod and  $\eta_4$  is the fraction of useful pump radiation power which is actually absorbed by the laser rod.

The extraction efficiency  $\eta_{extr}$  specifies which part of the excited power can be converted into laser radiation and coupled out of the resonator. The extraction efficiency and so laser output power depends on many parameters:

1. Resonator losses (scattering, diffraction).
2. Transmission of the output coupling mirror.
3. The ratio of the interacting volumes of both the radiation mode volume to the laser rod volume.

The extraction efficiency can be written as:

$$\eta_{extr} = \eta_5 \eta_6 \dots\dots\dots(2)$$

Where  $\eta_5$  is the output coupling efficiency (Koechner, 1996 and Yariv, 1989) and  $\eta_6$  is the spatial interaction efficiency SIE introduced by this paper. Finally, a very simple relationship is obtained for the total efficiency;

$$\eta_{tot} = \eta_1 \eta_2 \eta_3 \eta_4 \eta_5 \eta_6 \dots\dots\dots(3)$$

**THEORETICAL PROCEDURE**

The main idea optimizing the efficiency and output of the laser system is to design the resonator in such a way that the beam waist of the radiation mode within the laser rod is big enough to avoid spoiling of pumping energy stored in the active medium. To do so, we need to compute the beam volume within the laser rod as a function of rod diameter and rod length for a specified resonator dimensions.

To consider the influence of beam contraction (due to the geometrical designation and dimensions of resonator components) inside the laser resonator correctly, a parametric equation of the Gaussian beam contraction had been used. This equation introduce a minimum diameter  $2\omega_0$  at the beam waist where the phase front is plane. If the origin of z-coordinate is considered from this waist, the spot size a distance z from the beam waist expands as hyperbola, according to the relation (Koechner, 1996; Yariv, 1989).

$$\omega(z) = \omega_0 \left[ 1 + \left( \frac{\lambda z}{\pi \omega_0^2} \right)^2 \right]^{1/2} \dots\dots\dots(4)$$

Using the hyperbolic function given by Eq.(4) and with the help of schematic diagram shown in Fig. (1).

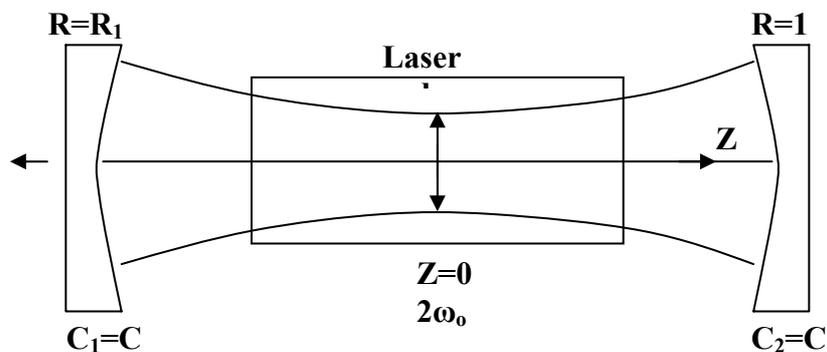


Fig. 1 : Model for the propagation of the radiation field in a resonator.

The volume of the mode radiation covered by z-depending spot size can be calculate as follows; starting with the most commonly used laser resonators, the resonator was assumed to be consisting of two spherical mirrors facing each other with radius of curvatures  $C_1=C_2=C$  and resonator length  $L$ .

The beam waist which occurs at the center of a symmetrical resonator configuration is given by (Koechner, 1996).

$$\left( 5 \omega_o^2 = \frac{\lambda}{2 \pi} [L (2 C - L)]^{1/2} \right)$$

By integrating the hyperbolic function  $\omega(z)$  for the boundaries shown in Fig. (1), an analytical derivation of the radiation mode (radiation field) volume had been done using Eq. (4) to get a relation illustrating the radiation mode volume  $V_M$  for the boundaries  $z = 0$  to  $z$ .

$$V_M = \lambda [L(2C - L)]^{1/2} Z + \frac{4\lambda}{3} [L(2C - L)]^{-1/2} Z^3 \dots\dots\dots(6)$$

For a cylindrical laser rod of length  $2z$  and radius  $r$ , the laser rod volume is given by:

$$V_p = 2 \pi Z r^2 \dots\dots\dots (7)$$

Now, the new correction factor  $\eta_6$ , that represents the ratio of radiation mode -volume to the total pumped volume of the active laser medium is given by:

$$\eta_6 = V_M / V_p$$

or

$$\eta_6 = \frac{D \lambda}{2 \pi r^2} \left[ 1 + \frac{4 Z^2}{3 D^2} \right] \dots\dots\dots (8)$$

where,

$$D = [L (2 C - L)]^{1/2} \dots\dots\dots (9)$$

In order to calculate the output power one generally uses (Koechner, 1996);

$$P_{out} = \eta_{tot} (P_{in} - P_{th}) \dots\dots\dots (10a)$$

or:

$$P_{out} = 0.5 I_s A \eta_5 \eta_6 (2 K P_{in} - L' + \ln R_1) \dots\dots\dots (10b)$$

Where  $\eta_{tot}$  is the total efficiency,  $P_{th}$  is the threshold pump power,  $L'$  being the constant nonoutput losses such as absorption in the active medium, absorption and scattering at the mirrors and diffraction losses of the resonator per round trip,  $K$  is the pumping coefficient,  $A$  is the cross sectional area of the rod,  $I_s$  is the saturation intensity and  $R_1$  is the reflectivity of the output mirror. For Nd:YAG,  $L'=0.075$ ,  $K=27 \times 10^{-6} \text{ W}^{-1}$  and  $I_s=920 \text{ W/cm}^2$  (Koechner, 1996 and Eicher, 1989).

Eq.(10b) can be readily differentiated with respect  $R_1$  and set the differentiation  $dP_{out}/dR_1$  equal to zero in order to determine the reflectivity  $R_1$  which gives maximum output power. Since the spatial efficiency factor  $\eta_6$  is independent on reflectivity  $R_1$ , as indicated by equations(8) and (9) and since.

$$\eta_5 = \frac{2(1 - R_1)}{(R_1)^{1/2} (L' - \ln R_1)} \dots\dots\dots (11)$$

The expression obtained for  $R_1$  should be similar to the expression given by (Koeckner, 1996).

$$R_1 \approx 1 - \frac{(2 K P_{in} L')^{1/2} - L'}{1 + L'} \dots\dots\dots (12)$$

**RESULTS AND DISCUSSION**

The Nd:YAG laser in this analysis was designed for a resonator consisting of two dielectrically coated mirrors of reflectivity  $R_1$  and  $R_2=1$  and of equal curvatures with  $C_1=C_2=C=100$  meters separated by a distance  $L=50$ cm. The corresponding beam within the resonator, consisted of a mode whose dimension was  $\omega_1 = \omega_2=0.13020$ cm at each mirror and constricted down to a diffraction-limited waist  $\omega_0=0.13013$ cm at the center of the resonator.

Considering the influence of radiation field interaction with laser rod, the above derived relation given by Eq.(8), which relate the radiation mode volume and the laser rod pumped volume through the spatial interaction efficiency factor was used to study the fractional decrease in efficiency due to incomplete overlapping effects, for the following two cases: Firstly, keeping the laser rod length constant, the SIE factor has been calculated as a function of laser rod radius. Secondly, keeping the laser rod radius constant, the SIE factor has been calculated as function of rod length as shown in Fig. (2 and 3).

Fig. (2), shows the variation of the SIE factor  $\eta_6$  as a function of laser rod radius, keeping rod length equal 20cm. The variation indicates that  $\eta_6$  is inversely proportional to  $r^2$ .

Fig. (3), illustrates the dependence of  $\eta_6$  on the laser rod length keeping rod radius constant at three independent values  $r_1=1.5$  mm,  $r_2=2$  mm and  $r_3=3$  mm .The three curves show an increase in  $\eta_6$  as rod length was increased.

Using Eq.(10b), an analytical calculation and qualitative comparison was made for the output powers as a function of input powers of 20cm long Nd:YAG laser rods of rod radius 1.5 mm and 2 mm.

Fig. (4), shows the output power versus lamp input power for four laser resonator configurations:

1.  $l=20$  cm,  $r=1.5$ mm,  $\eta_6=1$ ,  $R_1=0.7$
2.  $l=20$  cm,  $r=1.5$ mm,  $\eta_6=0.644$ ,  $R_1=0.7$
3.  $l=20$  cm,  $r=1.5$ mm,  $\eta_6=1$ ,  $R_1=0.85$
4.  $l=20$  cm,  $r=1.5$ mm,  $\eta_6=0.644$ ,  $R_1=0.85$

Fig. (5), again shows the output power versus lamp input power for another four laser resonator configurations:

1.  $l=20$  cm,  $r=2$ mm,  $\eta_6=1$ ,  $R_1=0.8$ .
2.  $l=20$  cm,  $r=2$ mm,  $\eta_6=0.423$ ,  $R_1=0.8$ .
3.  $l=20$  cm,  $r=2$ mm,  $\eta_6=1$ ,  $R_1=0.9$ .
4.  $l=20$  cm,  $r=2$ mm,  $\eta_6=0.423$ ,  $R_1=0.9$ .

We expect a larger diameter rod is able to capture a large fraction of pump radiation than the a small rod, but in fact only a small portion of the laser rod cross section will interact by the radiation field and lasing action occurred only in small volume of active material. Therefore the main reason for the poor overall efficiency is low interaction field mode by the small lasing volume. The data demonstrated on Fig. (4) and Fig. (5) give a comparison between resonator configurations with respect to output power and slope efficiency  $\eta_s$ . For maximized output power and higher slope efficiency an SIE factor of one is required i.e., for the same length and radius of laser rods, the slope efficiency of a laser which defined by  $\eta_s = dP_{out} / dP_{in}$ , is higher for configuration of  $\eta_6 = 1$  compared with configuration of  $\eta_6$  less them one.

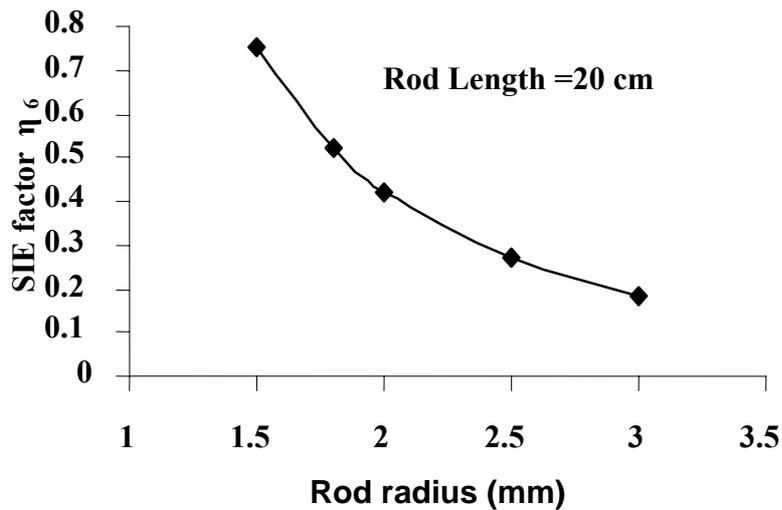


Fig . 2 : SIE factor ,  $\eta_6$  , for Nd:YAG laser with rod length 20 cm vs rod radius

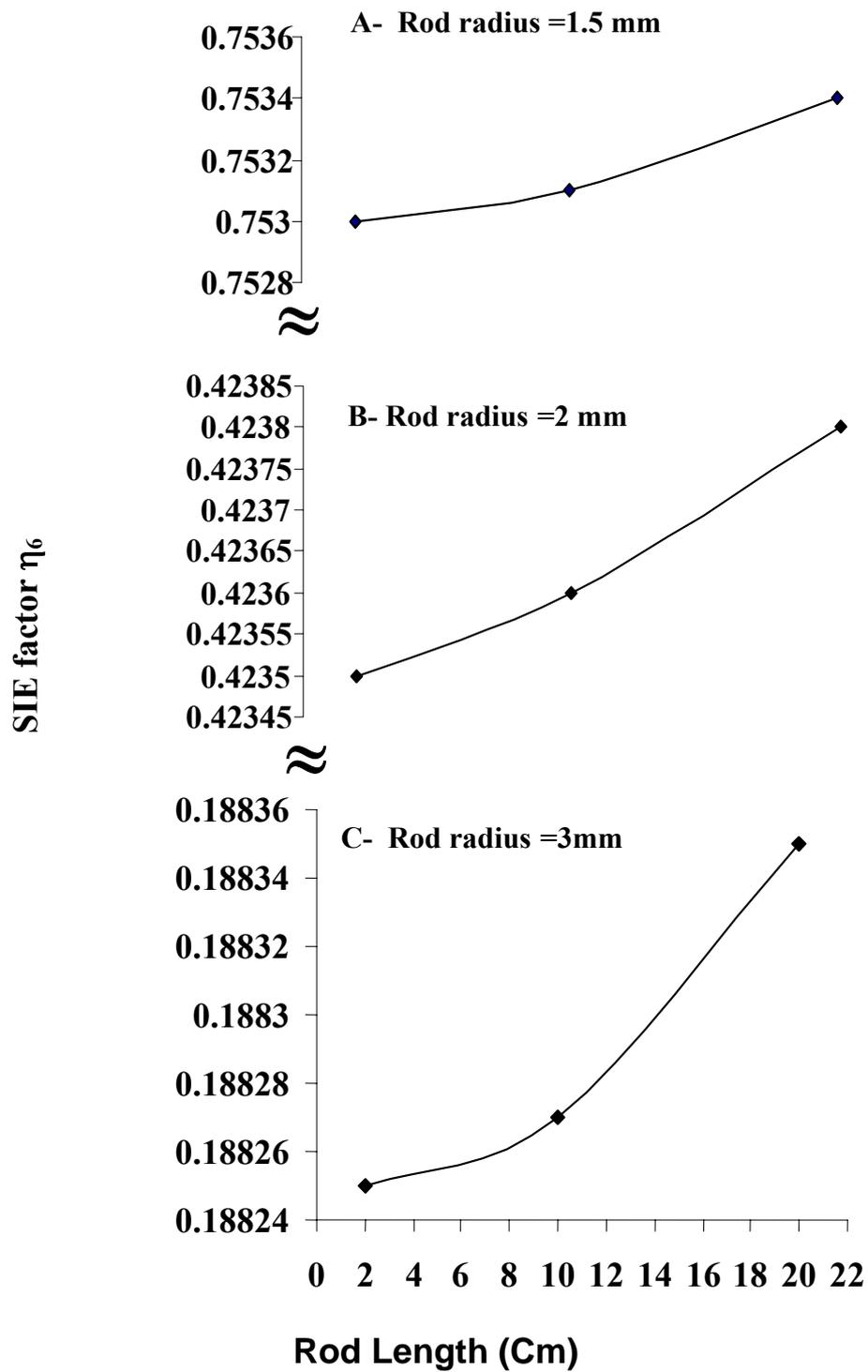


Fig. 3 : SIE factor for Nd :YAG laser with different radii vs rod length

A - Rod radius =1.5 mm

B- Rod radius = 2 mm

C- Rod radius =3mm

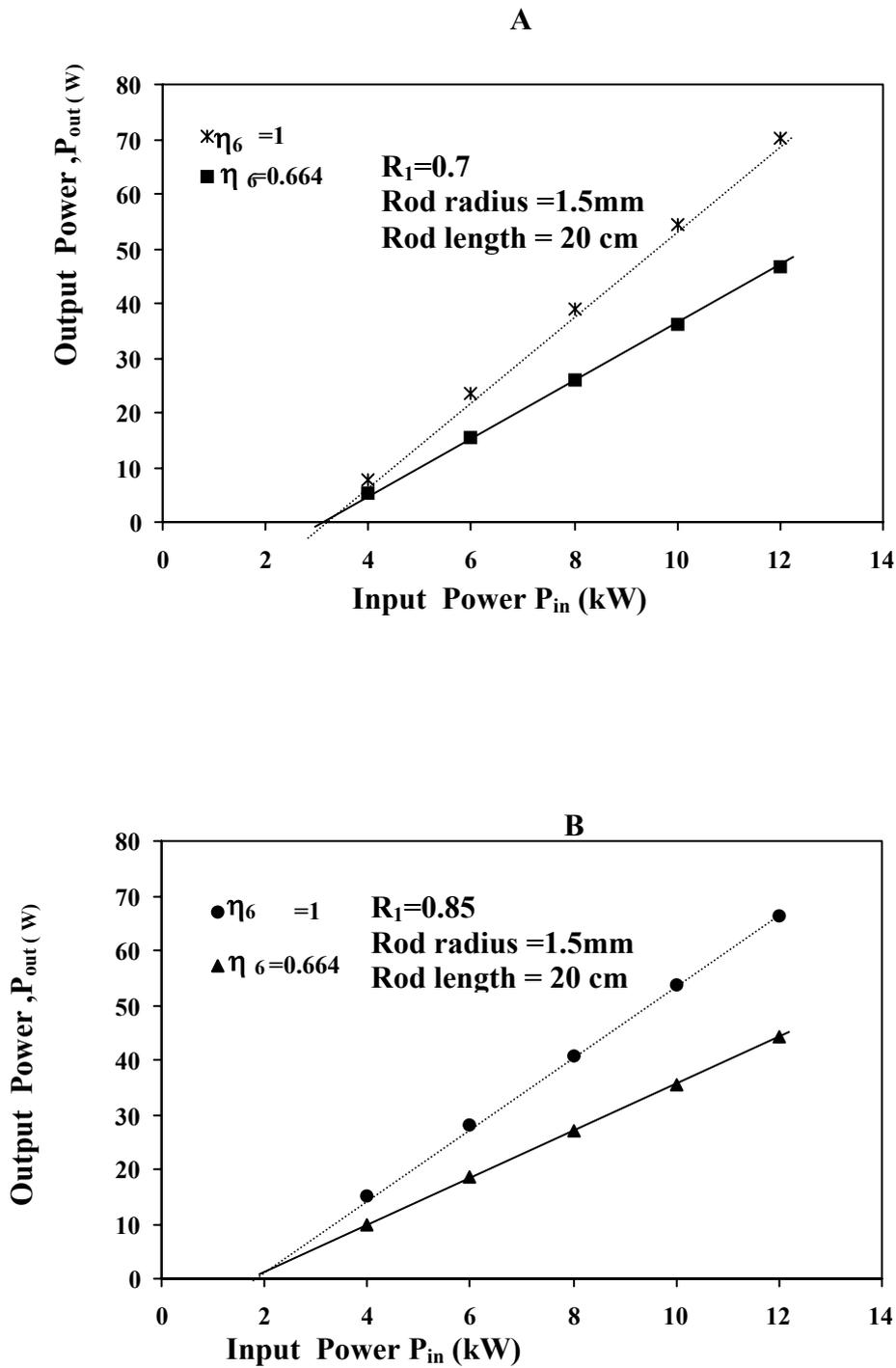


Fig. 4: Output power vs input power for Nd :YAG laser:

A :  $\eta_6 = 1$  ,  $R_1 = 0.7$  ,  $\eta_s = 0.78\%$   
 $\eta_6 = 0.664$  ,  $R_1 = 0.7$  ,  $\eta_s = 0.52\%$

B :  $\eta_6 = 1$  ,  $R_1 = 0.85$  ,  $\eta_s = 0.64\%$   
 $\eta_6 = 0.664$  ,  $R_1 = 0.85$  ,  $\eta_s = 0.43\%$

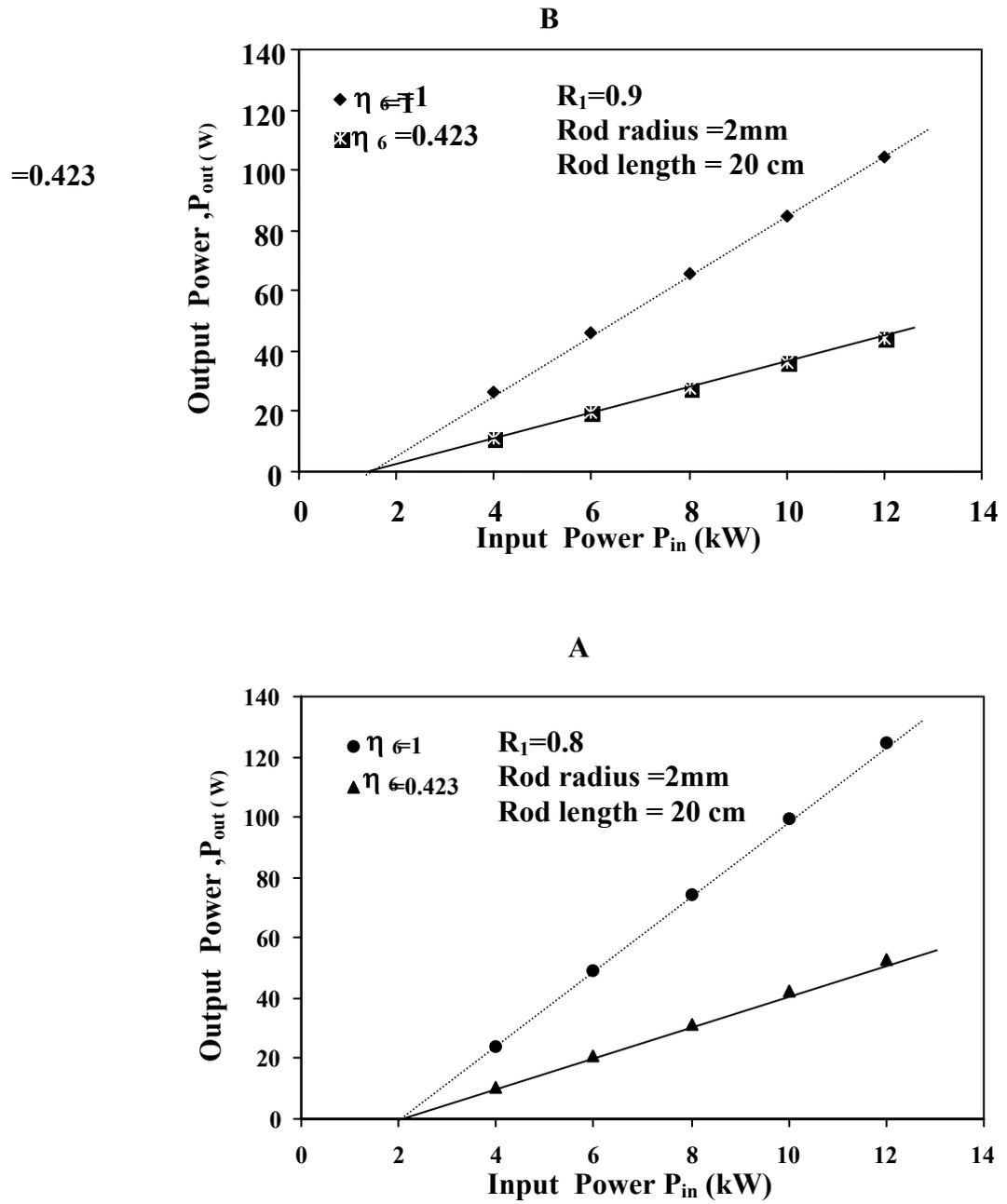


Fig. 5 : Output power vs input power Nd :YAG laser :

A :  $\eta_6 = 1$ ,  $R_1 = 0.8$ ,  $\eta_s = 1.26\%$ .

$\eta_6 = 0.423$ ,  $R_1 = 0.8$ ,  $\eta_s = 0.53\%$ .

B :  $\eta_6 = 1$ ,  $R_1 = 0.9$ ,  $\eta_s = 0.97\%$ .

$\eta_6 = 0.423$ ,  $R_1 = 0.9$ ,  $\eta_s = 0.41\%$ .

### CONCLUSION

Considering the influence of the radiation mode interaction and radiation beam construction, the relation of the total efficiency and output power of a Nd:YAG laser with spherically symmetric mirror resonator were derived. With these new parameters, a comparison between a laser system with SIE factor was made. It turned out that for SIE factor less than one and the same excitation power in both systems, a lower total efficiency and a lower output power. Therefore, the geometrical configuration and pumping arrangement have to be improved to realize the advantage of considering the above empirical relation.

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