

- ^{166,168,170}Hf

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- ¹⁶⁶⁻¹⁷⁰Hf

($2\theta / \hbar^2$)
 ($(\hbar^2 \omega^2) (I-2)$) (I)
 (E- GOS) (I) (E_γ / I)
 .X(5) ¹⁶⁶Hf SU(3)-O(6)
) (yrast)
 .(B(E2)
 . IBM-1

. - Hf :

Calculation of the Energies and the Reduced Electric Quadrupole Transition Probability of $^{166,168,170}\text{Hf}$ Even-Even Isotopes

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ABSTRACT

The property of $^{166-170}\text{Hf}$ even-even isotopes have been deduced, were the relation of the moment of inertia ($2\mathcal{I}/\hbar^2$) as a function of the square of the energy of the emitted photon when the nucleus translate from the (I) state to the ($I-2$) state ($\hbar^2\omega^2$) (back-bending curve) Also the relation of the gamma energy over spin (E_γ / I) as a function of the spin (I) (E-GOS) have been drawn, these curves indicated that these isotopes have a transition property (SU(3)-O(6), with the X(5) property for ^{166}Hf isotopes. To insure this property the calculation of the energy of different states in the yrast band has been done using the standard relation for the three types of nuclei depending on their property (rotation, γ -soft and vibration). Knowing the property of these isotopes, the calculation of electric quadrupole transition probability B(E2) has been done using Bohr and Mottelson model and the interacting boson model IBM-1 and compared with the experimental values.

Keywords: Energy Levels, Reduced Transition Probability, Hf even-even.

$$(2\mathcal{I}/\hbar^2)$$

$$(\hbar^2\omega^2)$$

(E-GOS) .(Sun, 2004)

(I) (E_γ / I)

(Regan *et al.*, 2003)

(Bohr and Mottelson, 1975)

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.(Krane, 1987)

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(Arima and Iachello, 1974) (IBM-1)

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SU(3)

O(6)

U(5)

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.(Arima and Iachello, 1987)

.(Iachello, 2001) E(5)

X(5)

(Abou-SalemandAbdElmageed ,2005)

(core-clusterdecomposition)

Hf

-

¹⁶⁶⁻¹⁸⁰Hf

(2006

)

¹⁷⁰⁻¹⁸⁰Hf

O(6)-SU(3)

¹⁶⁸Hf

¹⁶⁶Hf

SU(3)-O(6)

(IBM-1)

-

¹⁶⁸⁻¹⁷⁴Hf

(Doma *et al.*, 2011)

$$\frac{\hbar^2}{2\mathcal{G}}$$

¹⁷⁴⁻¹⁸⁰Hf

(Al-Maqtrayetal.,2011)

(IBM-1)

SU(3)-O(6)

-

¹⁶⁶⁻¹⁷⁰Hf

$$\frac{B(E_2, 4_1^+ \rightarrow 2_1^+)}{B(E_2, 2_1^+ \rightarrow 0_1^+)}$$

$E0_2^+ / E2_1^+$

$E4_1^+ / E2_1^+$

(E-GOS)

$$P = \frac{N_p \cdot N_n}{N_p + N_n}$$

(yrast)

$I - 2 \quad I$

:(Wong, 1990)

$$\frac{2\mathcal{G}}{\hbar^2} = \frac{4I - 2}{E(I) - E(I - 2)} = \frac{4I - 2}{E_\gamma} \dots\dots\dots (1)$$

$I - 2 \quad I \qquad\qquad\qquad (\hbar\omega)$

: [Sorensen,1973] $I - 2 \quad I$

$$\hbar\omega = \frac{E(I) - E(I - 2)}{\sqrt{I(I + 1)} - \sqrt{(I - 2)(I - 1)}} = \frac{E_\gamma}{\sqrt{I(I + 1)} - \sqrt{(I - 2)(I - 1)}} \dots\dots\dots(2)$$

(E-GOS)

(I) (E_γ / I)

$I \qquad\qquad\qquad E_\gamma$

:(Scharff-Goldhaber and Wensler, 1955)

$$E_\gamma(I \rightarrow I - 2) / I = \hbar\omega / I \dots\dots\dots(3)$$

$I \qquad\qquad\qquad E_\gamma$

:(Regan *et al.* , 2003)

Rotation $E_\gamma(I \rightarrow I - 2) / I = \frac{\hbar^2}{2\mathcal{G}} \times (4 - 2 / I) \dots\dots\dots(4)$

$I \qquad\qquad\qquad E_\gamma$

: (Bohr and Mottelson,1973)

$$E_\gamma(I \rightarrow I - 2) / I = \frac{E2_1^+}{4} (1 + 2 / I) \dots\dots\dots(5)$$

(1)

β

Q_0

:(Krane,1987)

.....

$$Q_0 = \frac{3}{(5\pi)^2} R_{av}^2 Z\beta(1+0.16\beta) \dots\dots\dots (6)$$

β

$$R_0 = 1.3 \text{ fm} \quad R_{av} = R_0 A^{1/3}$$

(Krane,1987)

$$\beta = \frac{4}{3} \frac{\sqrt{\pi}}{\sqrt{5}} \frac{\Delta R}{R_{av}} \dots\dots\dots (7)$$

$\Delta R < 0$

$\Delta R > 0$

$\Delta R = 0$

I

:(Wong,1990)

Q_0

$I - 2$

$$B(E_2; I \rightarrow I - 2) = \frac{15}{32\pi} e^2 Q_0^2 \frac{I(I-1)}{(2I+1)(2I-1)} \dots\dots\dots (8)$$

IBM-1

:[Arima and Iachello, 1987]

$$B(E_2; I + 2 \rightarrow I) = \alpha_2^2 \frac{3}{4} \frac{(I+2)(I+1)}{(2I+3)(2I+5)} (2N-I)(2N+I+3) \dots\dots\dots (9)$$

:[Arima and Iachello, 1987] (BE2)

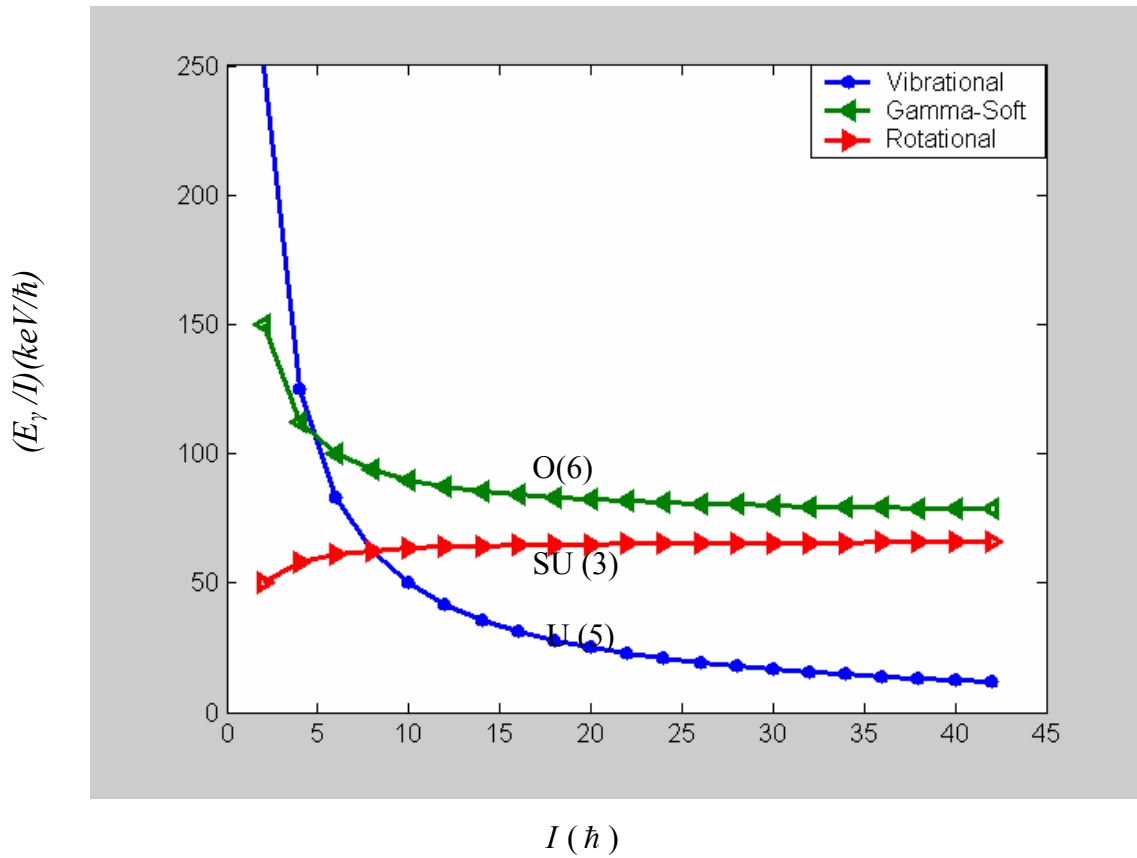
$$B(E_2; I + 2 \rightarrow I) = \alpha_2^2 \frac{(I+2)}{2(I+5)} \frac{1}{4} (2N-I)(2N+I+8) \dots\dots\dots (10)$$

α_2^2

I (

)

N



(Regan *et al.*, 2003)

(E-GOS) : 1

(94) - $^{166-170}\text{Hf}$ (72)

100.8 124.1 158.64 (2⁺) (98)

3.193 3.09 2.965 $\left(\frac{E_{4_1^+}}{E_{2_1^+}}\right)$ (keV)

I (29/ \hbar^2)

(2) (Wong, 1990) ($\hbar^2 \omega^2$) (I-2)

E- (I) (R = E_γ / I) (Regan *et al.*, 2003) (GOS)

(3) (

.....

$X(5)$
 (Clark *et al.*, 2003) $X(5)$

$^{166-170}Hf$
 ^{166}Hf
 .(1)

(Inan *et al.*, 2008) ^{166}Hf
 (2_1^+)

$X(5)$

$(3 \ 4 \ 5)$
 (2)

$(5 \ 4 \ 3)$

$(I-2)$

I

$B(E2)$

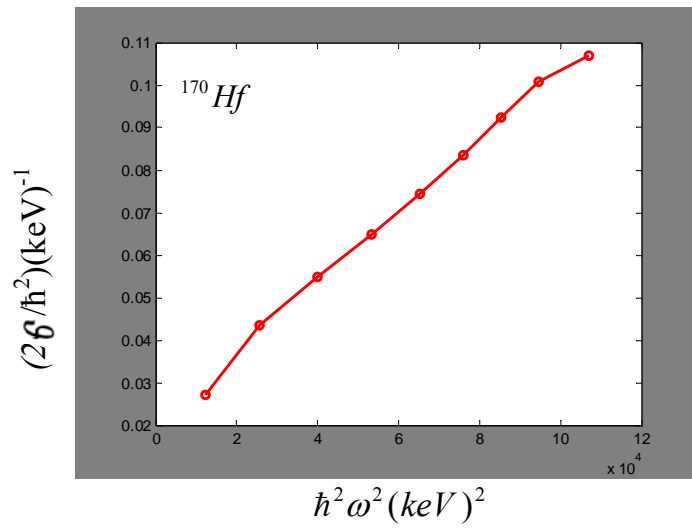
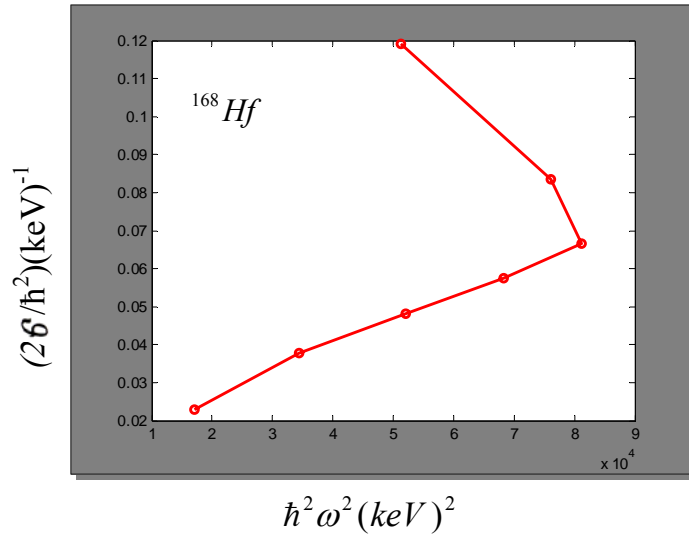
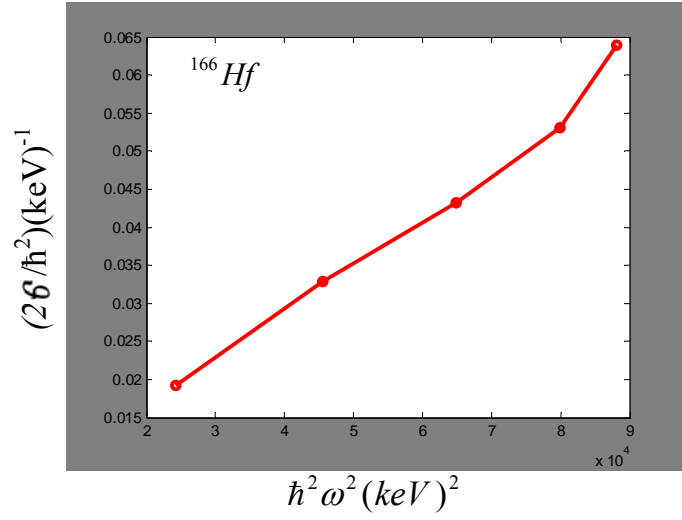
(IBM-1)

$X(5)$

(Bohr and Mottelson, 1973)

(Krane, 1987) $SU(3), O(6)$

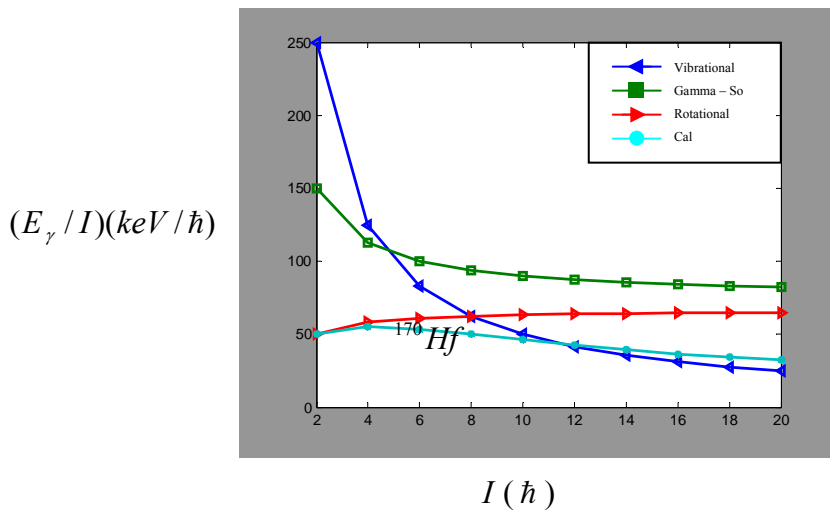
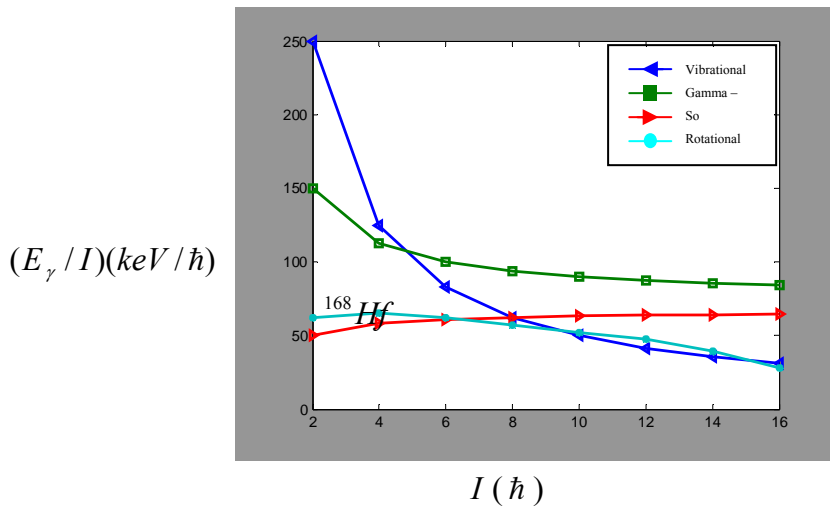
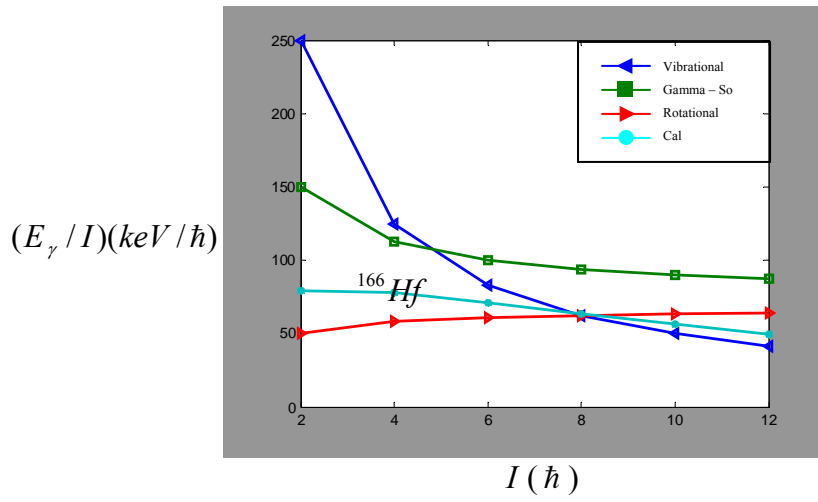
(Zamfir and Casten, 2003) ^{166}Hf
 . (8,7,6)



$^{166,168,170}\text{Hf}$

:2

.....



.(Bonatsos *et al.* , 2004) X(5) $^{166,168,170}\text{Hf}$

:1

	Experimental			X(5)
	^{166}Hf	^{168}Hf	^{170}Hf	
$E4_1^+ / E2_1^+$	2.96	3.11	3.19	2.91
$E0_2^+ / E2_1^+$	4.4	7.6	8.72	5.67
$\frac{B(E_2, 4_1^+ \rightarrow 2_1^+)}{B(E_2, 2_1^+ \rightarrow 0_1^+)}$	1.58	1.58	1.44	1.5
$P = \frac{N_p \cdot N_n}{N_p + N_n}$	5.45	5.83	6.15	5

(Inan *et al.* , 2008) $E2_1^+$

X(5)

:2

I^π	0	2	4	6	8	10	12
E	0	1	2.904	5.43	8.483	12.008	16.016

 ^{166}Hf

:3

I^π	Experimental	$E_{cal} (keV)$				
	(keV)* E_I	E_γ (keV)	Vibrator	$\gamma - Soft$	Rotor	X(5)
2_1^+	158.64	158.64	158.6	158.64	158.6	158.64
4_1^+	470.46	237.96	317.2	396.6	528.8	460.31
6_1^+	897.17	317.28	475.8	713.88	1110.5	859.51
8_1^+	1406.4	396.6	634.4	1110.48	1903.7	1343.23
10_1^+	1971.9	475.92	793	1586.4	2908.4	1904.98
12_1^+	2565.8	555.24	951.6	2141.64	4124.6	2540.8

* Ref.= (Baglin, 2008)

^{168}Hf

:4

I^{π}	Experimental	$E_{cal} (keV)$			
	$E_I (keV)^*$	$E_{\gamma} (keV)$	Vibrator	$\gamma - Soft$	Rotor
2_1^+	124.1	124.1	124.1	124.1	124.1
4_1^+	385.92	186.15	248.2	310.25	413.7
6_1^+	757.29	248.2	372.3	558.45	868.7
8_1^+	1213.7	310.25	496.4	868.7	1489.2
10_1^+	1736.06	372.3	620.5	1241	2275.2
12_1^+	2306.09	434.35	744.6	1675.35	3226.6
14_1^+	2857.5	496.4	868.7	2171.75	4343.5
16_1^+	3310.4	558.45	992.8	2730.2	5625.9

* Ref.= (Baglin, 2010)

 ^{170}Hf

:5

I^{π}	Experimental	$E_{cal} (keV)$			
	$E_I (keV)^*$	$E_{\gamma} (keV)$	Vibrator	$\gamma - Soft$	Rotor
2_1^+	100.8	100.8	100.8	100.8	100.8
4_1^+	321.9	151.2	201.6	252	336.0
6_1^+	642.9	201.6	302.4	453.6	705.6
8_1^+	1043.1	252.0	403.2	705.6	1209.6
10_1^+	1505.2	302.4	504.0	1008	1848.0
12_1^+	2016.1	352.8	604.8	1360.8	2620.8
14_1^+	2567	403.2	705.6	1764	3528.0
16_1^+	3151.3	453.6	806.4	2217.6	4569.6
18_1^+	3766.5	504.0	907.2	2721.6	5745.6
20_1^+	4421	554.4	1008.0	3276	7056.0

* Ref.= (Baglin, 2002)

^{166}Hf **B(E2)** :6

* $Q_0 = 6.514$ (Möller *et al.*, 1995) $\beta = 0.226$

$I \rightarrow I-2$	Experimental	$B(E2)_{cal}(W.u.)$			
	$B(E2)_{exp}(W.u.)^{**}$	Bohr and Mottelson	$IBM-1(O(6))$	$IBM-1(SU(3))$	X(5)
$2_1^+ \rightarrow 0_1^+$	128	155.72	161.27	128	128
$4_1^+ \rightarrow 2_1^+$	202	222.46	223.41	179.53	202.6
$8_1^+ \rightarrow 6_1^+$	280	250.48	255.91	190.12	290.66
$10_1^+ \rightarrow 8_1^+$	250	263.44	250	181.89	334.22
$12_1^+ \rightarrow 10_1^+$	155	286.11	234.58	168.29	-

* From equ (6) ** Ref.= (Baglin, 2008)

^{168}Hf **B(E2)** :7

* $Q_0 = 7.412$ (Möller *et al.*, 1995) $\beta = 0.254$

$I \rightarrow I-2$	Experimental	$B(E2)_{cal}(W.u.)$		
	$B(E2)_{exp}(W.u.)^{**}$	Bohr and Mottelson	$IBM-1(O(6))$	$IBM-1(SU(3))$
$2_1^+ \rightarrow 0_1^+$	154	198.40	206.169	154
$4_1^+ \rightarrow 2_1^+$	244	283.43	286.858	216.55
$6_1^+ \rightarrow 4_1^+$	285	312.17	322.140	231.78
$8_1^+ \rightarrow 6_1^+$	350	326.78	333.854	232.456
$10_1^+ \rightarrow 8_1^+$	370	335.64	330.400	225.094
$12_1^+ \rightarrow 10_1^+$	320	341.60	315.697	211.906
$14_1^+ \rightarrow 12_1^+$	240	345.87	291.820	193.846
$16_1^+ \rightarrow 14_1^+$	260	349.10	260	171.406

* From equ (6) ** Ref.= (Baglin, 2010)

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¹⁷⁰Hf

B(E2)

:8

* $Q_0 = 8.084$ (Möller *et al.*, 1995) $\beta = 0.274$

$I \rightarrow I - 2$	Experimental	$B(E2)_{cal} (W.u.)$		
	$B(E2)_{exp} (W.u.)^{**}$	Bohr and Mottelson	$IBM - 1(O(6))$	$IBM - 1(SU(3))$
$2_1^+ \rightarrow 0_1^+$	180	232.30	342.10	180
$4_1^+ \rightarrow 2_1^+$	260	331.86	477.66	253.70
$6_1^+ \rightarrow 4_1^+$	258	365.51	539.22	272.70
$8_1^+ \rightarrow 6_1^+$	300	382.61	562.9	275.20
$10_1^+ \rightarrow 8_1^+$	310	392.99	562.63	268.9
$12_1^+ \rightarrow 10_1^+$	280	399.96	544.89	256.45
$14_1^+ \rightarrow 12_1^+$	300	404.97	513.11	238.85
$16_1^+ \rightarrow 14_1^+$	≈ 330	408.74	469.28	216.72
$18_1^+ \rightarrow 16_1^+$	≈ 320	411.68	414.64	190.36
$20_1^+ \rightarrow 18_1^+$	≈ 350	414.04	350	159.96

*From equ (6)

** Ref.= (Baglin, 2002)

- ¹⁶⁶⁻¹⁷⁰Hf

$(E4_1^+ / E2_1^+)$ $(E2_1^+)$

¹⁶⁶Hf

.O(6)- SU(3)

$(R = E4_1^+ / E2_1^+ = 2.9)$ $(E2_1^+ = 158.64 keV)$

¹⁶⁶Hf

$(\hbar^2 \omega^2)$

$(2.9 / \hbar^2)$

(2)

(8_1^+)

(I)

$(R = E_\gamma / I)$

(E-Gos)

(3)

I

O(6) SU(3)

(1)

X(5)

O(6)- SU(3)

.X(5)

(3) X(5) X(5)

($2_1^+ \rightarrow 0_1^+$)

SU(3) O(6) (12₁⁺ → 10₁⁺)

α_2 (6) X(5)

($\alpha_2^2 = 2.327$) ($2_1^+ \rightarrow 0_1^+$) SU(3) α_2

($\alpha_2^2 = 4.887$) ($10_1^+ \rightarrow 8_1^+$) O(6) α_2

(E-Gos) α_2

. O(6)- SU(3)

X(5)

(12₁⁺ → 10₁⁺) (10₁⁺ → 8₁⁺)

O(6) SU(3)

O(6) ($10_1^+ \rightarrow 8_1^+$) SU(3) ($2_1^+ \rightarrow 0_1^+$)

α_2

($E2_1^+ = 100.8 keV$) ¹⁷⁰Hf ($E2_1^+ = 124.1 keV$) ¹⁶⁸Hf

¹⁷⁰Hf ($R = E4_1^+ / E2_1^+ = 3.109$) ¹⁶⁸Hf

O(6)- SU(3) ($R = E4_1^+ / E2_1^+ = 3.19$)

(10₁⁺) ¹⁶⁸Hf (2) SU(3)-O(6)

(yrast)

¹⁷⁰Hf

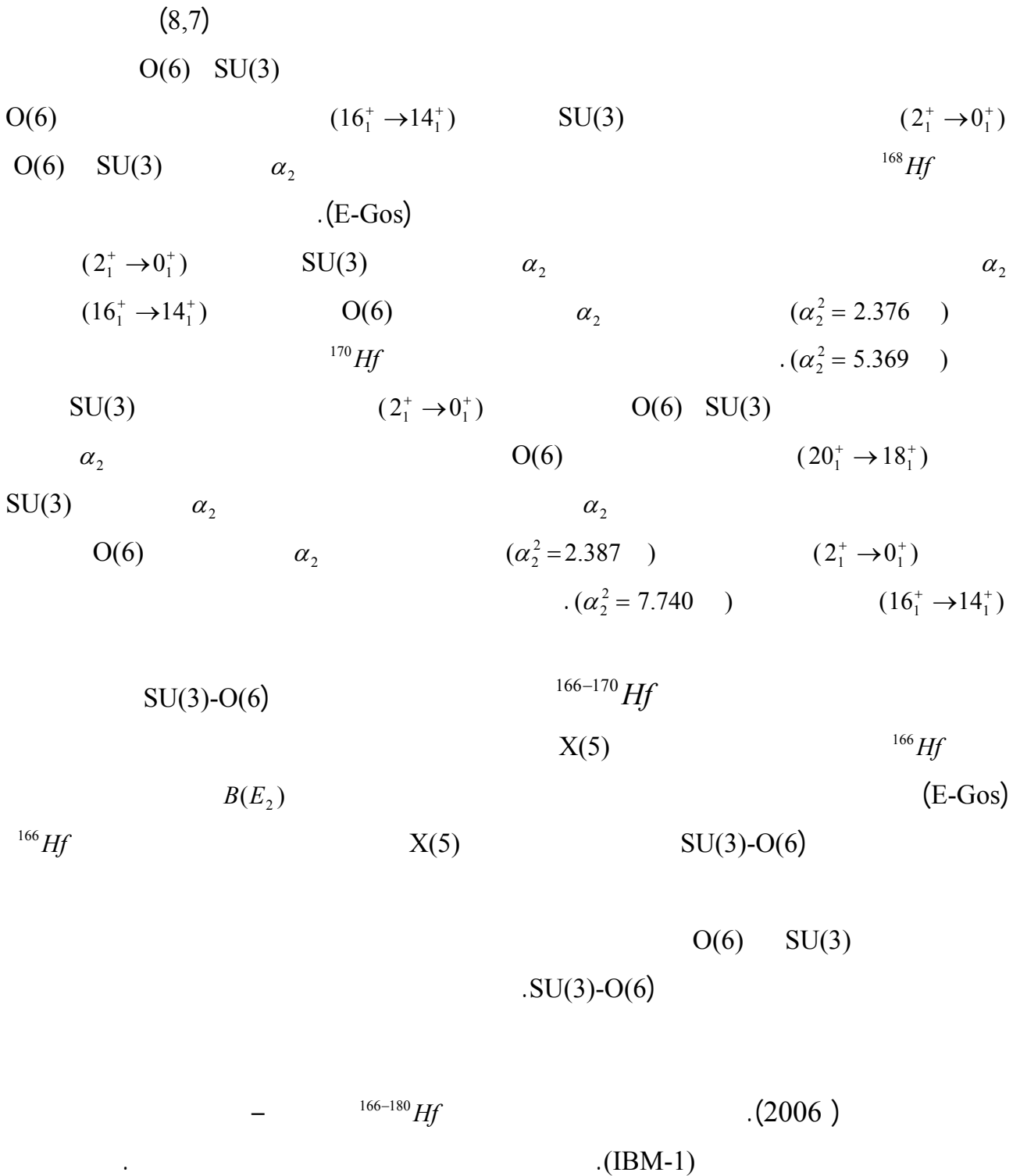
(E-Gos) ($16_1^+ \rightarrow 4_1^+$)

¹⁷⁰Hf ¹⁶⁸Hf (3)

(5 4) O(6) SU(3)

SU(3) O(6)

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