Application of Natural Reaeration Kinetics to Artificial Aeration

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(Received 9/11/2006, Accepted 1/10/2007)

ABSTRACT

The recalibrated Thackston and Krenkel (TK) equation, which was originally derived for natural reaeration of streams and rivers, is used here for the evaluation of artificial reaeration of water. As many as 180 experiments were conducted in order to provide the data needed for this study. A laboratory – scale open channel was used for the experimental part of this study. Artificial roughness was provided by attaching crushed stones to the channel bed. Each of the three sets of experiments involved was performed using different size of bed stone, namely; 12.70 mm, 22.75 mm and 34.60 mm. The bed slopes used at each set of experiments were (0.005, 0.004, 0.003, 0.002, 0.001 and 0.0005 ), respectively.

An average calibration constant (C) of \( (218 \times 10^{-6}) \) was obtained for TK equation for artificial reaeration. In terms of Froude number (F), the calibration constant (C) was found to have a value of \( (249 \times 10^{-6} F^{0.22}) \). The analysis of the data obtained reveals that the reaeration coefficient \( (k_2) \) is a function of channel bed slope. The higher values of the ratio \( (u^*/h) \) were obtained at the lower values of shear velocities \( (u^*) \) and flow depth \( (h) \). At each slope in each set of experiments the relationship between \( (k_2) \) and \( (u^*/h) \) was found to be of power function.
The dissolved oxygen (DO) content of both water and wastewater is a significant quality parameter. In surface waters it is vital for fish life and other aquatic biota. It is also a viable factor in self-purification phenomenon which naturally takes place in waste laden streams and other water sheds. Besides, DO concentration in municipal and industrial wastewaters is an indicative characteristic for the oxygen consuming contents of such waters.

Reaeration is the only applicable mean by which water may regain its DO content. Generally, in surface waters such as streams, lakes, and impounded reservoirs, the atmospheric reaeration is the natural process through which waters suffering oxygen deficit could recover the DO concentration.

The atmospheric reaeration has already been the subject of a considerable number of papers since the late 1950’s, starting from that of O’ Connor and Dobbins (1958). Almost all of such works had focused on mechanism and kinetics of atmospheric reaeration in flowing streams. (Churchill et al., 1962), (Thackston and Krenkel, 1969), (Parkhurst and Pomeroy, 1972), (Wilcock, 1988), (Genereaux and Hemond, 1992), (Weber and DiGiano, 1996), (Melching and Flores, 1999), (Gualtieri and Gualtieri, 2000), (Thackston and Dawson, 2001) and (Gualtieri et al., 2002) are examples for that trend.

The parameters affecting the natural reaeration process in streams were a controversial subject among different researchers. Careful inspection of equations in the literature shows that mass – transfer at the air – water interfaces has been affected by 14 different parameters (Gualtieri et al., 2002). Key parameters include; mean water depth, mean flow velocity, Froude number, gravitational acceleration, bed slope, shear velocity, discharge, Manning’s roughness and others.

However, these parameters can be deduced to small number of parameters. At a fixed temperature, the dimensionless reaeration rate was finally found to be a function of only Froude number, channel slope, Reynolds number and relative roughness. (Gualtieri et al., 2002).

In this context, a number of predictive equations have been introduced for the atmospheric reaeration. Nevertheless, application of the available equations is limited to the field data for which each equation was developed.

Moog and Jirka (1998) had shown that some of the commonly used equations were not very accurate. Meanwhile some other equations were said to be satisfactorily
accurate. Thackston and Krenkel is one of such atmospheric reaeration prediction equations. This equation was later recalibrated by Thackston and Dawson (2001) against field data and found to be one of the most accurate predictive equations. Reviewing the pertinent literature reveals that reaeration analysis efforts have been limited to natural reaeration in streams and rivers. In many cases practical applications need an enhanced aeration of waters such as in wastewater treatment facilities. Artificial reaeration is also frequently required to promote oxidation of certain materials contained in some waters or to accelerate the process of reoxygenation in waters that suffer dissolved oxygen deficit. For this reason the research work presented in this paper was undertaken to investigate to what extent the recalibrated Thackston and Krenkel equation which was originally derived for natural stream reaeration, may fit with the experimental data obtained in the work.

The data involved in this paper were obtained through using a Japanese made laboratory steel channel with (12 m) length, (0.5 m) width and (0.5 m) depth. The glass sides of the channel enables precise measurements of flow depth using the point gauge. Water is pumped through the channel with a centrifugal pump of (100 L/s) capacity. Discharge through the channel was measured with a sharp edged v-notch submerged weir. Flow velocity was measured using an accurate laboratory current meter as well as with the v-notch weir. Water temperature was continuously recorded.

The channel bed was covered with stones in order to increase bed roughness. The stones used were fixed on the channel bed with tar in order to prevent washout. Standard sieving was used for the preparation of these bed stones.

The total number of the experiments performed was (180). These (180) experiments were divided into three sets. The first set of experiments was fulfilled with stone bed of average size (D50) of (12.70 mm). The second set was conducted using stone bed of average size of (22.75 mm), while this size was (34.60 mm) in the third set of experiments. In each set of experiments the slope of the channel was changed 6 times. These slopes have the values of (0.005, 0.004, 0.003, 0.002, 0.001 and 0.0005), respectively. For each slope the flow rate was changed 10 times.

### THE EQUATION USED

As pointed out earlier the equation involved in this paper is the recalibrated Thackston and Krenkel equation, henceforth will be referred to as TK equation. Moog and Jirka (1998) have reviewed the relevant literature for the accuracy of reaeration equations. After an intensive analysis they concluded that TK equation is one of the most accurate reaeration equations and it has the advantage of the dimensional homogeneity as well. The original form of TK equation is:

\[
 k_2 = C \left( \frac{u^*}{h} \right) \quad \ldots (1)
\]

Where \( k_2 \) = reaeration coefficient, \( C \) = calibration constant, \( u^* \) = shear velocity of flow, \( h \) = water mean depth.

The calibration constant \( C \) was found to have the following relationship with Froude number (Gualtieri et al., 2002).

\[
 C = 0.000125 \left(1 + F^{1/2}\right) \quad \ldots (2)
\]

Where \( F \) = Froude number.
Later on TK equation was recalibrated against field data (Thackston and Dawson, 2001) and a new relationship between C and F was introduced: The new relationship has the form of;

\[ C = 0.000025 (1 + 9 F^{1/4}) \]

\[ \text{………………(3)} \]

Accordingly, the recalibrated TK equation became:

\[ k_2 = 0.000025 (1 + 9F^{1/4}) \frac{u^*}{h} \]

\[ \text{………………(4)} \]

This form of the recalibrated TK is used in this paper.

**RESULTS AND DISCUSSION**

As previously mentioned, the first set of experiments was performed using a stone bed of an average particle size of (12.70 mm). The stone average size for the second set of the experiment was (22.75 mm), while for the third set of the experiments this size was (34.60 mm).

The stone was provided to the channel bed to develop fully turbulent flow by bottom shear. This was one of the bases on which the original TK equation was built.

The results of application of TK equation to the data obtained in the first set of experiments are shown in Figures (1, 4, 7 and 10). The results of application of TK equation concerning the second set of experiments are given in Figures (2, 5, 8 and 11), where the remaining Figures (3, 6, 9 and 12) are those of the third set of experiments.

The figures are arranged in the order shown so that to reveal the effect of the stone size on the reaeration coefficient.

Each of Figures (1, 2 and 3) are plotted using the whole data points obtained in the corresponding set of experiments, regardless of bed slope. This is in order to show the effect of bed stone size on the reaeration coefficient. Generally, for the shear velocities used, it seems that the reaeration coefficient \((k_2)\) decreased as the bed stone size was increased.

![Fig.1: k2 versus (u*/h) for bed stone size D50 =12.70mm](image-url)
As shown in Figures (1 and 2), the statistical regression analysis yielded a calibration constant (C) of \((220.3 \times 10^{-6})\) for the first set of experiments and \((220.32 \times 10^{-6})\) for the second set of experiments, with \((R^2)\) values of \((0.9790)\) and \((0.9936)\), respectively. For the third set of experiments, (Fig. 3), a value of \((C)\) of
(213.67 $\times 10^{-6}$) was obtained, with ($R^2$) of (0.9936). The average ($C$) value will be ($218 \times 10^{-6}$). Hence, the form of TK equation that fit the operational conditions of this study will be:

$$k_2 = 218 \times 10^{-6} \left( \frac{u^*}{h} \right)$$

The effect of increasing the slope of channel bed is shown in Figures (4, 5 and 6). It is apparent, in the three sets of experiments, that increasing the bed slope has significantly increased the reaeration coefficient ($k_2$). This is simply due to the increase in shear velocity as the bed slope was increased. On the other hand, it is clear that for each bed slope in each of the three sets of experiments, the higher values of reaeration coefficient ($k_2$) were obtained at the higher values of the ratio ($u^*/h$). These high ($u^*/h$) values were recorded at both; the lower shear velocity and flow depth values. The data recorded suggest that decreasing the depth of flow is more effective in enhancing reaeration of waters.

Table 1: Relationship between $k_2$ and ($u^*/h$) for the different slopes in the first set of experiments (bed stone size = 12.70 mm).

<table>
<thead>
<tr>
<th>Slope of channel bed</th>
<th>Relationship</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.005</td>
<td>$k_2 = 218.52 \left( \frac{u^*}{h} \right) 0.8278$</td>
<td>0.9989</td>
</tr>
<tr>
<td>0.004</td>
<td>$k_2 = 209.65 \left( \frac{u^*}{h} \right) 0.811$</td>
<td>0.9962</td>
</tr>
<tr>
<td>0.003</td>
<td>$k_2 = 200.57 \left( \frac{u^*}{h} \right) 0.8078$</td>
<td>0.9977</td>
</tr>
<tr>
<td>0.002</td>
<td>$k_2 = 189.93 \left( \frac{u^*}{h} \right) 0.8121$</td>
<td>0.9994</td>
</tr>
<tr>
<td>0.001</td>
<td>$k_2 = 129.33 \left( \frac{u^*}{h} \right) 0.7404$</td>
<td>0.9869</td>
</tr>
<tr>
<td>0.0005</td>
<td>$k_2 = 166.25 \left( \frac{u^*}{h} \right) 0.8638$</td>
<td>0.9961</td>
</tr>
</tbody>
</table>
Table 2: Relationship between $k_2$ and $(u^*/h)$ for the different slopes in the second set of experiments (bed stone size = 22.75 mm).

<table>
<thead>
<tr>
<th>Slope of channel bed</th>
<th>Relationship</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.005</td>
<td>$k_2 = 215.69 \ (u^*/h) \ 0.8423$</td>
<td>0.9989</td>
</tr>
<tr>
<td>0.004</td>
<td>$k_2 = 202.68 \ (u^*/h) \ 0.7824$</td>
<td>0.9942</td>
</tr>
<tr>
<td>0.003</td>
<td>$k_2 = 200.24 \ (u^*/h) \ 0.8337$</td>
<td>0.9965</td>
</tr>
<tr>
<td>0.002</td>
<td>$k_2 = 189.21 \ (u^*/h) \ 0.8261$</td>
<td>0.9963</td>
</tr>
<tr>
<td>0.001</td>
<td>$k_2 = 184.72 \ (u^*/h) \ 0.8732$</td>
<td>0.9969</td>
</tr>
<tr>
<td>0.0005</td>
<td>$k_2 = 186.45 \ (u^*/h) \ 0.8188$</td>
<td>0.9981</td>
</tr>
</tbody>
</table>

Fig. 5: $k_2$ versus $(u^*/h)$ for bed stone size $D_{50} = 22.75\ mm$

Fig. 6: $k_2$ versus $(u^*/h)$ for bed stone size $D_{50} = 34.60\ mm$
Table 3: Relationship between \( k_2 \) and \( \frac{u^*}{h} \) for the different slopes in the third set of experiments (bed stone size = 34.60 mm).

<table>
<thead>
<tr>
<th>Slope of channel bed</th>
<th>Relationship</th>
<th>( R^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.005</td>
<td>( k_2 = 195.66 \left( \frac{u^*}{h} \right)^{0.7287} )</td>
<td>0.9896</td>
</tr>
<tr>
<td>0.004</td>
<td>( k_2 = 188.79 \left( \frac{u^*}{h} \right)^{0.7616} )</td>
<td>0.9967</td>
</tr>
<tr>
<td>0.003</td>
<td>( k_2 = 173.83 \left( \frac{u^*}{h} \right)^{0.7003} )</td>
<td>0.9738</td>
</tr>
<tr>
<td>0.002</td>
<td>( k_2 = 171.47 \left( \frac{u^*}{h} \right)^{0.7782} )</td>
<td>0.9918</td>
</tr>
<tr>
<td>0.001</td>
<td>( k_2 = 158.38 \left( \frac{u^*}{h} \right)^{0.7886} )</td>
<td>0.9937</td>
</tr>
<tr>
<td>0.0005</td>
<td>( k_2 = 155.43 \left( \frac{u^*}{h} \right)^{0.8411} )</td>
<td>0.9963</td>
</tr>
</tbody>
</table>

The relationships between Froude number (F) and \( \frac{k_2}{u^*/h} \) for the three sets of experiments are shown in Figures (7, 8 and 9). These figures obviously show the strong relationship between the corresponding variables. The nearly same equation obtained indicate that the bed stone size has no effect on the form of the relationship between Froude number and the ratio \( \frac{k_2}{u^*/h} \). Taking the average value of the exponent shown in the resulting power equations, the form of the relationship between (F) and \( \frac{k_2}{u^*/h} \) will be:

\[
k_2 \left( \frac{u^*}{h} \right) = 249 \times 10^{-6} F^{0.22} \quad \ldots(6)
\]

The means that the calibration constant (C) in terms of Froude number is:

\[
C = 249 \times 10^{-6} F^{0.22} \quad \ldots(7)
\]

Fig.7: \( k_2/(u^*/h) \) versus Froude Number for bed stone size \( D_{50} = 12.70 \text{mm} \)
Fig. 8: $k_2/(u^*/h) \times 10^{-6}$ versus Froude Number for bed stone size $D_{50} = 22.75$ mm

Fig. 9: $k_2/(u^*/h) \times 10^{-6}$ versus Froude Number for bed stone size $D_{50} = 34.60$ mm

Figures (10, 11 and 12) were constructed in order to show the effect of bed slope on the relationship between Froude number ($F$) and the ratio $k_2/(u^*/h)$. Inspecting these figures clearly show that, except for the first set of experiments where $D_{50}$ is 12.70 mm this relationship did not considerably affected by bed slope.
Fig. 10: $k_2 / (u^*/h)$ versus Froude Number for bed stone size $D_{50} = 12.75\text{mm}$

Fig. 11: $k_2 / (u^*/h)$ versus Froude Number for bed stone size $D_{50} = 22.75\text{mm}$

Fig. 12: $k_2 / (u^*/h)$ versus Froude Number for bed stone size $D_{50} = 34.60\text{mm}$
CONCLUSIONS

The experimental works fulfilled in this study followed by the analysis of the data obtained confirm the following conclusions:

1. Natural reaeration kinetics of streams and rivers can successfully be applied to artificial reaeration of water.
2. The artificial reaeration coefficient \( (k_2) \) decreases when the stone size of the flow channel bed increases.
3. Increase in the channel bed slope significantly increase the artificial reaeration coefficient \( (k_2) \).
4. For a constant bed slope the reaeration \( (k_2) \) increases as the ratio \( (u^* / h) \) increases.
5. At the lower values of shear velocity and depth of flow, a higher values were recorded for the ratio for shear velocity to depth of flow \( (u^* / h) \).
6. Decreasing the depth of flow is more effective than increasing shear velocity in enhancing reaeration of water.

REFERENCES